

Force chains and the fragmentation of granular materials

Luis E. Vallejo, Zamri Chik, & Sebastian Lobo-Guerrero
*Department of Civil and Environmental Engineering, University of Pittsburgh,
Pittsburgh, Pennsylvania, USA*

Bernardo Caicedo
*Department of Civil and Environmental Engineering, Universidad de los Andes,
Bogotá, Colombia*



ABSTRACT

In this study, the fragmentation of gravels is evaluated by uniaxial compression tests and by numerical and theoretical analyses. The Discrete Element Method (DEM) was used for the numerical analysis. Fractal theory was used to interpret the laboratory results. The laboratory and the DEM results indicated that the gravel under compression developed force chains in order to resist the uniaxial compressive loads. When the intensity of the force chains exceeded the uniaxial compressive strength of the individual pieces of gravel, the gravel broke into smaller pieces. The resulting grain size distribution was found to be fractal in nature.

RESUMEN

En este estudio, el fracturamiento de gravas es evaluado usando ensayos de compresión unidireccional, por simulaciones numéricas, y por un análisis teórico. Las simulaciones numéricas usan el Método de Elementos Discretos (DEM). La teoría de los fractales fue usada para interpretar los resultados de laboratorio. Los experimentos y las simulaciones numéricas indicaron que la grava bajo compresión desarrolla cadenas de fuerza entre las partículas. Estas cadenas de fuerza resisten la fuerza de compresión inducida a la grava. Cuando la intensidad de las cadenas de fuerza excede la resistencia a la compresión de la grava, esta se quiebra. La distribución del tamaño de granos después de la compresión resulta ser fractal.

1 INTRODUCTION

Granular materials forming part of civil engineering structures such as rockfill dams and the granular base in pavement systems are subjected to sustain compressive stresses resulting from gravity and traffic loads respectively. As a result of these compressive stresses, the granular materials break into pieces of different sizes. Thus, the original engineering properties with which the rockfill dam or the base of a pavement structure was designed (i.e., hydraulic conductivity, shear strength, elastic moduli) will change during its engineering life. Changes in the original engineering properties could affect the stability of the structures and could make them unsafe. The size distribution of the broken granular material has been found to be fractal in nature (McDowell, et al., 1996; Perfect, 1997). However, there is not a clear explanation to date about the mechanisms that cause the granular materials to develop a fractal size distribution. In the present study, a test designed to crush granular materials and its simulation using the Discrete Element Method (DEM) are developed to understand the mechanisms involved with the development of a fractal fragmentation by granular materials.

roundness number $R = 0.5$, and for the rounded gravel, $R = 0.8$ (Krumbein, 1941). The average diameter for both gravels, $d_{50} = 7$ mm, and their specific gravity, G_s , was equal to 2.67. Both gravels were classified as GP using the Unified Soil Classification System.

A uniaxial compression experiment was carried out in a tube with a 5 cm interior diameter and a height of 14 cm. The gravel filled the tube up to a height equal to 10 cm (Figure 1). For meaningful test results, it is found

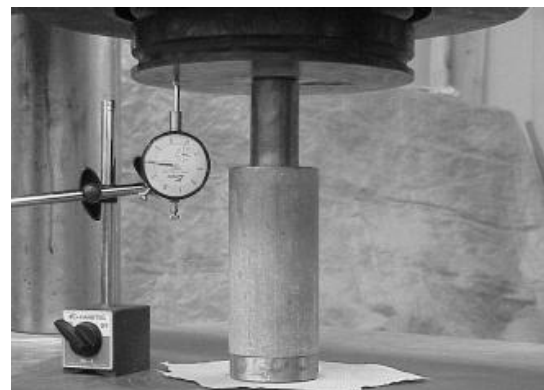


Figure 1. Equipment used to crush the gravels

2 LABORATORY EXPERIMENTS

2.1 Materials and Equipment Used

For the laboratory experiments round and angular gravel were used. The angular gravel had a Krumbein's

necessary to maintain a ratio of sample diameter to the maximum particle size of approximately 6:1 or greater. The tube was set up right with a steel plug at the bottom

on which the gravel rest and a 2.0 kg piston head pressing against the top of the gravel. The objective of the exercise was to impound a crushing condition to the grains and to investigate the crushing characteristics of the particles. The induced crushing load was performed using a Universal Testing Machine commonly used in most laboratories however, with the addition of a piston that could fit inside the test cylinder extending to the top of the test specimen.

The gravel was loaded to three different uniaxial compressive stresses. These stresses were equal to 10.0 MPa, 11.8 MPa and 18.2 MPa. Due to the position of the individual grains and the consequent manner by which the particle-to-particle contact points developed and changed throughout the tests, varying contact forces were experienced by individual grains in the specimen.

As the uniaxial compressive stress was applied to the gravel grains, they experience abrasion and crushing. The abrasion and crushing occurred all over the particulate system without any particular pattern. With the crushing of a particle, a sudden addition of voids was created since the broken particles substituted solid particles in the system. With crushing, the point-to-point contact between the particles was reduced as the amount of fragments increased. How a sample of gravel looked at the end of a uniaxial compression test can be seen in Figure 2.

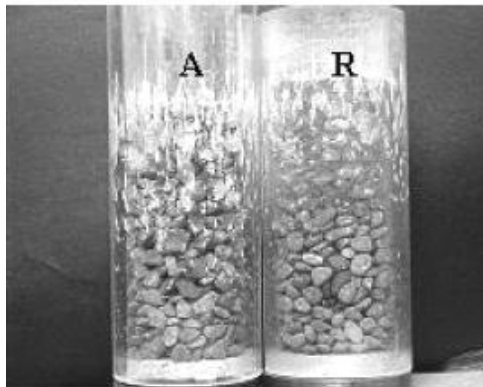


Figure 2. Crushed and intact angular (A) and rounded (R) gravels after compression test

After the completion of the uniaxial compression test, the gravel samples were removed from the tubes and a sieve analysis was performed on them. The sieve analysis was made on each of the samples removed from the tubes that were made of broken and unbroken grains. The results of the sieve analysis produced particle size distributions that were used to determine the fragmentation fractal dimension, D_F , of the samples (mentioned in Section 2.2). The results of the sieve analysis are shown in Figures. 3 and 4. These figures indicate that the samples upon crushing developed a particle size distribution that was well graded or fractal in nature. The samples experienced a higher level of fragmentation with an increase in the value of the uniaxial compressive stress.

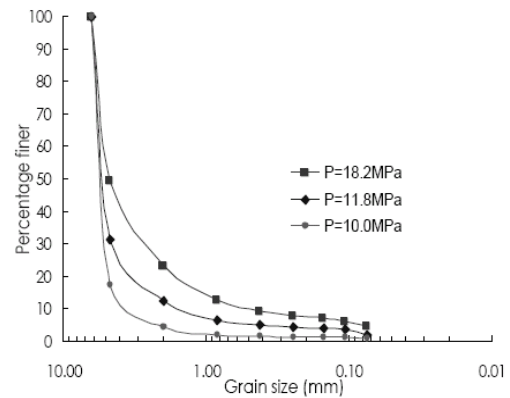


Figure 3. Particle size distribution after crushing tests for round (R) gravels

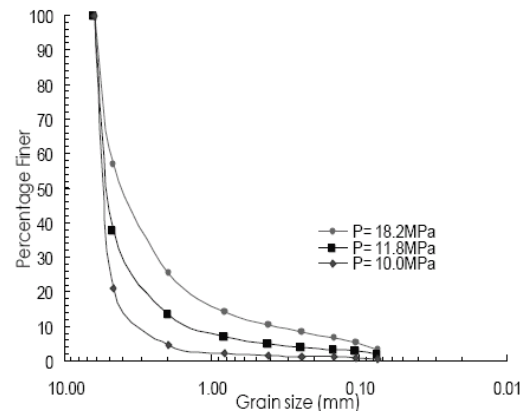


Figure 4. Particle size distribution after crushing for angular (A) gravels.

2.2 Fragmentation Fractal Dimension of the Grain Size Distribution

Particle size distribution of naturally occurring soils have been found by Tyler and Wheatcraft (1992) and Hyslip and Vallejo (1997) to be fractal. According to Tyler and Wheatcraft (1992), the particle size distribution in a natural soil can be obtained using the following equation:

$$N(R > r) = kr^{-D_F} \quad [1]$$

where $N(R > r)$ is the total number of particles with linear dimension R (radius of the particle) which is greater than a given size r ; k is a proportionality constant; and D_F is the fractal dimension of the size distribution of grains. As a result of compression, the size distribution in a granular soil will change. Changes in the size distribution of the grains will be reflected in the values of D_F . Thus, grain

fragmentation in soils subjected to compressive stresses can be evaluated by the changes in their fragmentation fractal dimension, D_F .

It is very time consuming to apply the number-based relationship expressed by Equation 1, is very time consuming. Another relationship that uses the results of a standard sieve analysis test was developed by Tyler and Wheatcraft (1992) to calculate the fragmentation fractal dimension, D_F , of natural soils. This relationship is:

$$\frac{M(R < r)}{M_T} = \left(\frac{r}{r_L} \right)^{3-D_F} \quad [2]$$

where $M(R < r)$ is the cumulative mass (weight) of particles with size R smaller (finer) than a given comparative size r ; M_T is the total mass (weight) of particles; r is the sieve size opening; r_L is the maximum particle size as defined by the largest sieve size opening used in the sieve analysis; and D_F is the fragmentation fractal dimension. The results of a sieve analysis tests using Equation 2 can be plotted on log-log paper. The slope, m , of the best fitting line through data obtained using Equation 2 and the fractal dimension, D_F , are related as follows (Tyler and Wheatcraft, 1992):

$$D_F = 3 - m \quad [3]$$

Equations 2 and 3 were used to obtain the fractal dimension of the size distribution of gravel subjected to crushing in the uniaxial compression tests (Figures 1 and 2). The fragmentation fractal dimension, D_F , for the grain size distributions shown in Figures 3 and 4 are shown in Figures 5, 6 and 7.

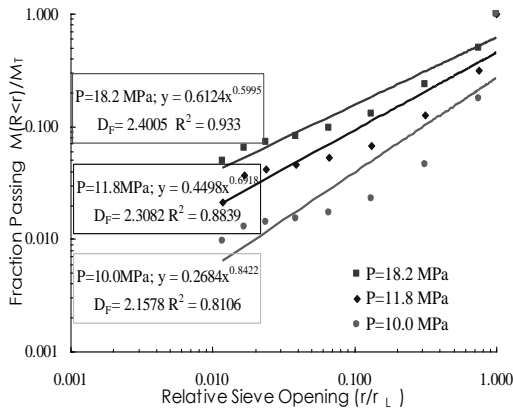


Figure 5. Fragmentation of round (R) gravels and the related fragmentation fractal dimension D_F

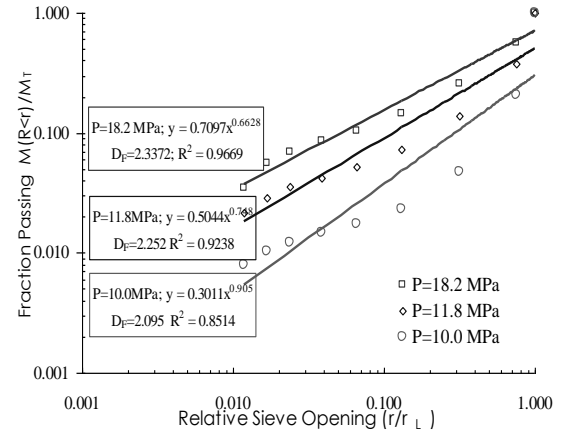


Figure 6. Fragmentation of angular (A) gravels and the related fragmentation fractal dimension D_F

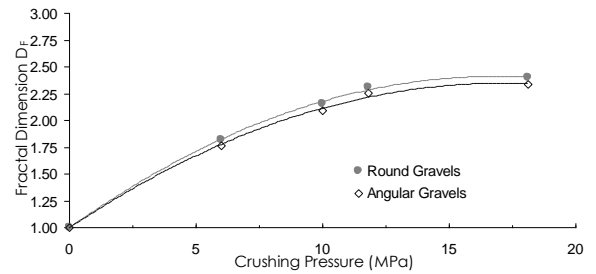


Figure 7. Relationship between the fragmentation fractal dimension D_F and the applied crushing pressure for both the rounded (R) and angular (A) gravels.

An analysis of Figures 5, 6 and 7 indicates that the fragmentation fractal dimension, D_F increases with the uniaxial compressive stress applied to the gravel. The fragmentation fractal dimension measures the degree of crushing of the gravel. The greater the fragmentation fractal dimension, the greater is the level of breaking of the gravel particles.

3 AN EXPLANATION FOR THE DEVELOPMENT OF A FRACTAL SIZE DISTRIBUTION IN THE SAMPLES

Next, an explanation why the gravel developed a fractal size distribution is carried out using the Discrete Element Method. The PFC^{2D} code developed by Itasca (202) was used in order to gain an understanding how the gravel grains interact, distribute the reacting forces and resist the uniaxial compressive loads

3.1 Previous DEM Simulations on Crushing

Besides laboratory tests, there is another tool that can be used to analyze crushing in granular materials. Numerical simulations in the form of the Discrete Element Method (DEM) can also be used parallel to laboratory tests to allow a better visualization of the crushable behavior of granular materials. Originally developed by Cundall and Strack (1979), DEM has been used to simulate the behavior of granular assemblies when subjected to different loading conditions. Commercial codes such as the Itasca PFC^{2D} are based on this method. However, the PFC^{2D} does not allow particle breakage.

Different solutions have been proposed in order to overcome the constraint of no particle breakage when working with DEM codes. One solution to this problem is to treat each granular particle as a porous agglomerate built by bonded uniform smaller particles. In order to establish a failure criterion, the strength of the bonding between the particles forming the agglomerates can be specified. This approach has been used by Jensen et al. (2001).

Another different solution to the particle breakage problem is to replace the particles that are fulfilling a predefined failure criterion with an equivalent group of smaller particles. Tsoungui et al. (1999) used this approach considering that a particle fulfilling a predefined tensile failure criterion could be replaced by a group of eight particles. This latter approach was used by Lobo-Guerrero et al. (2006) to simulate crushing in granular materials under static uniaxial compression tests. In this study, the work by Lobo-Guerrero et al. (2006) was used.

3.2 Configuration of the Simulated Material and Testing Procedure

The first step in the compression simulation was the generation of the sample. A simulated box was created using the PFC^{2D} program which is based on DEM (Itasca Consulting Group, 2002). This simulated box container measured 0.05 m in width and 0.10 m in height. The coefficients of normal and shear stiffness of the walls were set to 1×10^9 N/m, and their friction coefficient was set to 0.7. After this, 120 particles having a radius of 3 mm were randomly generated inside the box with the constraint of no overlaps between them. The density of these particles was set to $2,500 \text{ kg/m}^3$. Their friction coefficient was also set to 0.7. These particles were allowed to settle under a 1g gravity field ($1g = 9.81 \text{ m/sec}^2$). For the static uniaxial compression tests, compression was induced in the sample by a vertically moving piston plate with a velocity equal to 0.0625 mm/sec. The piston plate applied a load to the particles that varied from $1 \times 10^4 \text{ N}$ to $1 \times 10^5 \text{ N}$.

3.3 Particle Breakage Criterion

When using the PFC^{2D} program, particles are idealized as discs that interact with each other at their contacts. This interaction is mainly governed by three models: the stiffness, the slip, and the bonding models. Only the first two models were used in this study. The failure criterion for the particles when subjected to compressive loads has been presented and discussed in detail by the authors in

previous publications (Lobo-Guerrero et al., 2006; Vallejo et al., 2006). This failure criterion is based on the forces acting on a particle, its coordination number, and its size. A summary of the failure criterion is as follows: (a) only particles with coordination number equal or less than 3 are allowed to break. When a particle breaks, it is replaced by eight smaller particles of different sizes. Of these eight, two have a radius equal to 0.5 times the radius of the unbroken particle, two have a radius equal to 0.333 times the radius of the unbroken particle, and four have a radius equal to 0.167 times the radius of the unbroken particle, (b) the type of loading exerted to a particle with a coordination number equal or smaller than three is similar to the one exerted on a particle by a Brazilian type of test (Fairhurst, 1964), and (c) the tensile stress at which a particle breaks is equal to: $3 \times 10^6 [r]^{-1} \text{ Pa}$, where r is the radius of the particle. For a particle with a 3 mm radius used for the testing program, the tensile stress to break it into eight particles is equal to 10^6 Pa (Figure 8). The process of breaking is a continuous process, however, as the particles get smaller, the tensile stress needs to increase in order to break the particles as stated by the relationship between the tensile stress and r .

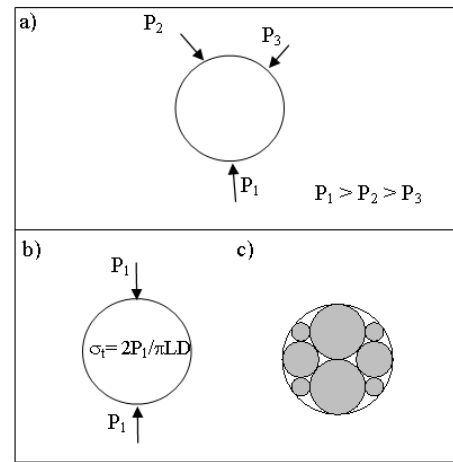


Figure 8. The induced tensile stress and the produced fragments after failure

3.4 Results of the Static Compression Tests

After the granular material was placed in the box, it was subjected to compressive loads that varied in value from $1 \times 10^4 \text{ N}$ to $1 \times 10^5 \text{ N}$. Figure 9(a) shows the particles when subjected to a compressive load equal to $1 \times 10^4 \text{ N}$, Figure 9(b) shows the particles subjected to $3 \times 10^4 \text{ N}$, and Figure 9(c) shows the particles subjected to $1 \times 10^5 \text{ N}$. Figure 9(a) shows no sign of particle breakage. Figures 9(b) and 9(c) show a large portion of the original particles experienced fragmentation as a result of the increased compressive loads. These figures also indicate that the location of the broken particles were uniformly distributed throughout the sample when the axial loads were large to produce particle's breakage (Figure 9(c)). Also, Figure 9 indicates that the reaction of the particles to the applied

axial loads was through the development of force chains at contact points between the particles. The intensity of these force chains is related to the thickness of the force chains in the figure. The thicker the force chains are, the larger the interparticle forces are (Figure 9).

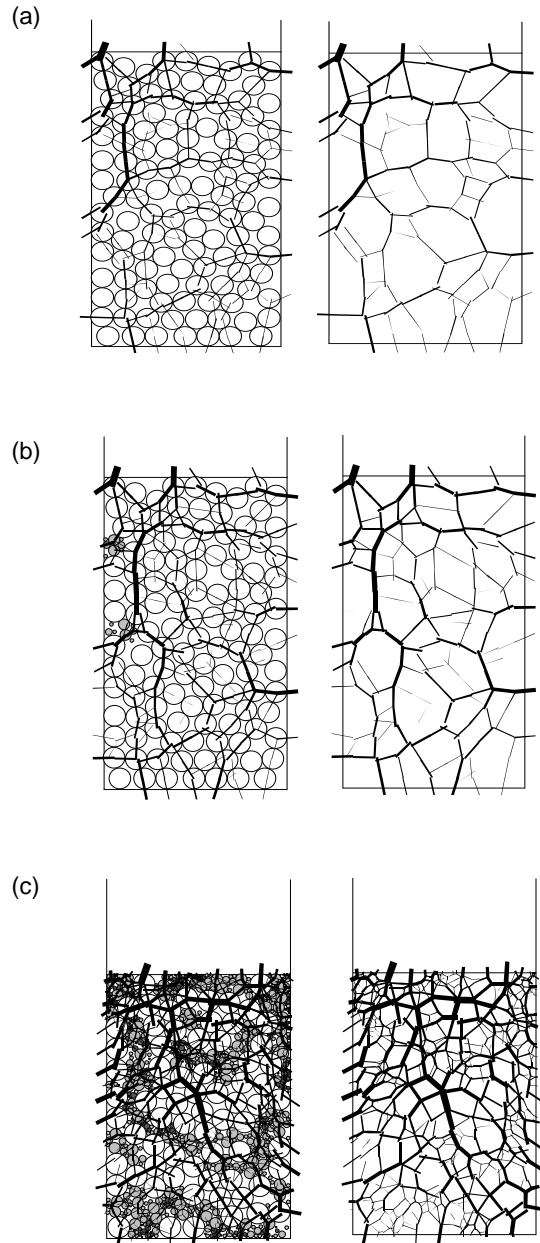


Figure 9. (a) Particle crushing and force chains in the samples subjected to: (a) $1 \times 10^4 \text{ N}$, (b) $3 \times 10^4 \text{ N}$, and (c) $1 \times 10^5 \text{ N}$ compressive loads.

3.5 Fractal Analysis of the Network of Force Chains

An analysis of the force chains developed by the disks indicates that these force chains are distributed like a network in the granular system and the network consists of branches of varying degrees of intensity (the thicker the force chain in Figure 9, the larger the force). Using the box method from fractal theory, the fractal dimension of the network was obtained. The box method uses grids made of squares of different sizes that are placed on top of the force networks [Figure 9(b)]. If one plots in a log-log paper the number of boxes intercepted by the force chains versus the size of the squares, one obtains the fractal dimension of the distribution, D_D , of the force chains in Figure 9 (b). This has been done in Figure 10 for the force chain shown in Figure 9(b). The value of $D_D \approx 1.3$ (which is the absolute value of the exponent of the variable x)

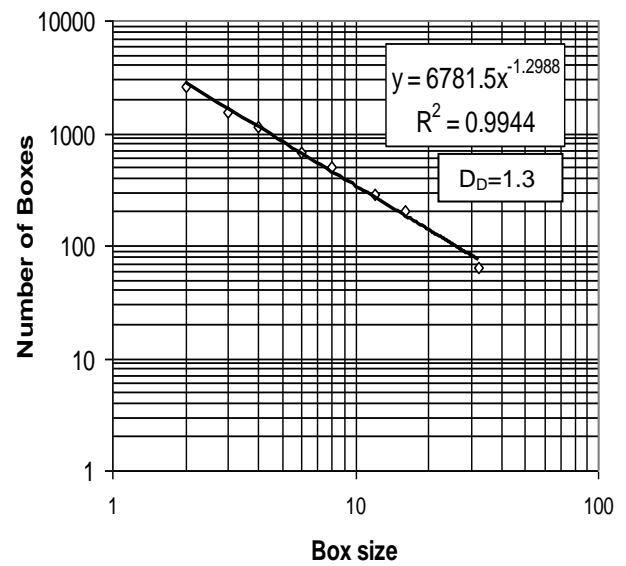


Figure 10. The fractal dimension of the distribution of force chains in Fig. 9(b)

An analysis of Figure 10 indicates that the distribution of the force chains in Figure 9(b) is indeed fractal. The fractal dimension of the distribution of forces in the granular system, D_D is equal to $1.2988 \approx 1.3$.

The intensity of the forces in the chains shown in Figure 9(b) was also obtained using the PFC^{2D} code. This code has a subroutine that creates a contact pointer that goes contact by contact calculating and classifying the resultant force in each contact. At the end of the run of this subroutine, the number of the contacts with their respective contact forces are obtained. Using this information, a log-log plot of the number of contacts, N , with a force, R , greater than certain value r is plotted against the contact force, r . The result of this analysis is shown in Figure 11.

An analysis of the results shown in Figure 11 indicates that the distribution of the intensity of force chains in the network shown in Figure 9(b) is also fractal. The fractal

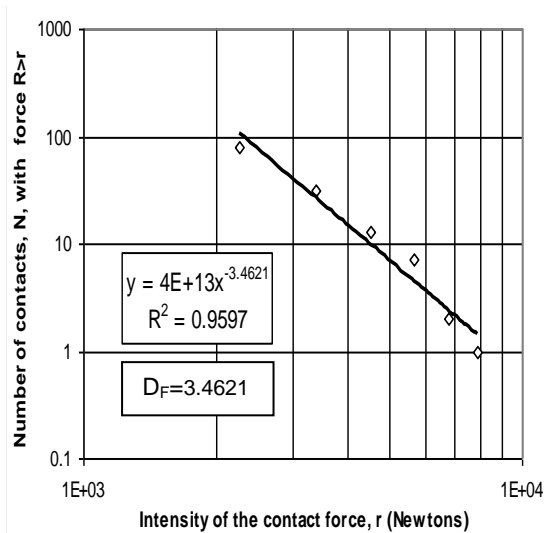


Figure 11. Plot to obtain the fractal dimension of the force intensity distribution, $D_F = 3.4621$

dimension of the distribution of the intensity of force chains in the network shown in Figure 9(b) was found to be equal to 3.4621 (Figure 11).

3.6 Discussion of the Results

An analysis of the results shown in Figures 9, 10 and 11 indicates that the distribution of the network of force chains as well as the distribution of their intensities are fractal in nature. Because the distribution of the force chains does not cover all the grains, some of the grains are subjected to the force chains while some of the grains are not. In fact if one looks at Figure 9, some of the idle grains (the ones that not carry any load), can be removed without affecting the stability of the granular system.

The distribution of the intensity of the force chains was also found to be fractal (Figure 11). Thus, some of the particles will be subjected to complete crushing [Figures 9(b) and 9(c)]. Some of the particles will be partially broken under the moderate loads and some of the particles will resist the load without breaking (the white particles in Figure 9). Thus, the end result of the compression process will be a system of granular material that is fractal on their size distribution as shown by the laboratory experiments (Figures 5 to 7).

4. CONCLUSIONS

Laboratory compression tests on a relatively uniform granular material (fine gravel) and their simulation using the Discrete Element Method indicated the following:

- (1) The effect of a compressive load exerted on the fine gravel was to break the particles into a new system that had a size distribution that was fractal in nature.
- (2) The cause for the fractal size distribution of the broken and unbroken grains was a network of force chains that was fractal in its intensity and distribution.

5. ACKNOWLEDGMENTS

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